Stochastic Optimization and Pension Fund Management

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11th International Conference on Stochastic Programming (SPXI).
Vienna, Austria. August 2007.

In this tutorial

How to implement a multi-stage stochastic pension fund management system considering regulatory and organizational risk constraints for an arbitrary country from the scratch (just in case you have to) in three steps:

- Step One: Specify the needs for successful Pension Fund Management (PFM), and clarify its relation to Asset Liability Management (ALM).
- Step Two: Analyze current implementations and integrations of PFM approaches into multi-stage stochastic optimization frameworks.
- Step Three: Combine and adapt suitable implementations and apply it to a specific country, considering the respective underlying particularities (in this case for Austria).

Step One

- Specify needs for successful Pension Fund Management (PFM)
- Clarify relation to classical Asset Liability Management (ALM)
- Define modeling issues of underlying Assets and Liabilities

Retirement and Long-Term Investments

Retirement and Pension Systems

- Pay-as-you-go: direct re-distribution / inter-generation contract
- **Fully funded: wealth accumulation / pension fund or insurance**
- Individual saving: individual strategy (life-styling) / tax advantages

Goals of fully funded pension funds are to

- fulfill promised (DB)/expected (DC) pension payments to every member,
- fulfill legal restrictions and diversification requirements,
- invest administered funds systematically,
- distribute surpluses transparently,
- reflect interests of all involved parties.

Pension Plans and Risk Management

Pension plans are available in two varieties:

- Defined benefit (DB) pension plans: benefits for plan participants are specified in advance, e.g. a percentage of the final or average salary (indexation). The plan sponsor bears the investment risk.
- Defined contribution (DC) pension plans: The benefits that accrue remain uncertain, and depend on contributions as well as the development of invested funds.

Major advantage of DB plans over DC arrangements is in the benefit stability provided to employees, but the recent pension crisis was (clearly) due to DB plans.

Pension Funding Crisis and Modeling (Fabozzi et al., 2004)

At the end of 2002 more than 90% of the private-sector defined-benefit pension plans in the UK and the US were underfunded. Frequently cited reasons:

- Prolonged contribution time-out due to rising equity markets followed by the sharp drop of stock markets in the year 2000.
- Lower interest rates which raised the value of liabilities.
- Business problems faced in industries (automobile, airlines, and steel).
- Actuarial/accounting practices allowed the underfunding to go undetected.

Conclusion: In many cases, bad modeling (or absence of modeling) of this ALM problem helps to explain the pension funding crisis.

Selection of the type of Pension Plans

Sweeting (2007) surveyed all FTSE100 companies and concluded the type of pension arrangement chosen by a sponsoring employer appears to say a lot about that firm, e.g.

- difference between industries, e.g. Information Technology and Resources firms less likely offer a DB pension scheme, and
- unprofitable firms tend to avoid offering DC pensions (flexibility of DB pension contributions is appreciated by firms making less profit).

Mixture contracts are also available, e.g. DB-type during active phase, and DC-type during passive phase.

Note: Each pension plan type needs a different modeling approach.

Differences to classical ALM - Competing Interests

Pension Fund Manager must decide on investment strategy, surplus distribution, and legal reporting, given stakeholders competing interests:

- Investors / Pension Fund sponsoring company:
  - Reduce necessity for additional sponsoring.
- Young active members
  - Flexibility and lowest possible contributions,
  - high returns from investment strategy.
- Old active members and passive members (pensioners)
  - Secure and safe pension, i.e. low risk investment strategy.

Differences to classical ALM - Assets & Liabilities

Larger set of assets and liabilities compared to classical Bank ALM systems, e.g. (Kusy and Ziemba, 1986) - furthermore both depend on different (correlated) underlying factors.

Liabilities

- Expected future benefit payments (mortality, entry, retirement, disability)
- Accrual and Valorization (wage increase, inflation)

Assets

- (Classical) asset returns
- Contributions by employer and participants

The importance of the liability side

- Growing awareness of liability risk: focus shifting from asset returns to an integrated view of assets and liabilities (adoption of liability benchmarking).

- Ability to effectively hedge liability risk heavily depends on:

  1. a correct projection of liabilities, and
  2. an understanding of correlations between assets and liabilities.

Full-fledged, integrated ALM is the only chance to to keep pace with liabilities growth, especially in low-return environments. Importance of risky alternative positions (e.g. hedge funds) growing, rarely above 10% of total asset portfolio.

Longevity (LifeMetrics Technical Document 1.0, 2007)

Longevity is a high-profile risk for DB pension plans:

- Appropriate update of mortality tables (used to calculate pension liabilities) $\rightarrow$ significant increase in the value of liabilities $\rightarrow$ substantial widening of pension deficits.
- On average, each additional year of life adds approximately 3-4% to the value of UK pension liabilities.
- (Pension Capital Strategies and Jardine Lloyd Thompson, 2006) study of FTSE100 companies: assumptions about mortality rates used in pension valuations were overly optimistic. Realistic longevity assumptions would more than double the aggregate deficit from GBP 46 billion to GBP 100 billion.

Asset Management

"Okay, this time we'll try it *your* way: 3% in stocks, 2% in bonds and 95% in lottery tickets."

Asset Management

Asset allocation of main asset categories differs significantly between country and date of observation, and is constantly evolving. Also subject to regulatory constraints, and heavily depending on the respective sponsor in some cases:

	Netherlands		Switzerland		United Kingdom		United States	
	1992	2001	1992	2001	1992	2001	1992	2001
Cash & deposits	1.9	1.5	10.0	8.5	3.6	3.3	4.5	3.7
Bonds	22.8	34.7	40.5	35.9	9.9	14.5	31.1	23.1
Equities	17.8	49.5	13.1	39.0	74.8	63.5	46.5	59.8
Loans	48.3	8.8	34.8	13.8	0.1	0.0	2.8	1.8
Other	9.2	5.4	1.6	2.9	11.6	18.8	15.0	11.5

(Fabozzi et al. 2004)

Step Two

- Summarize existing methodologies for handling long-term PFM
- Outline conceptional and numerical issues for solving multi-stage stochastic optimization problems
- Survey selected multi-stage stochastic PFM optimization frameworks

Decision Support Models for Strategic ALM

- Simulation (flexible, simple, most commonly chosen)
  - Decisions are input $\longrightarrow$ *static* decision making.
- Single-stage Risk-Return Optimization under Uncertainty:
  - Static decisions are computed, various risk measures are supported (Markowitz-style), but only a short- or mid-term time horizon can be considered.
- Robust Optimization - recent development, e.g. (Iyengar and Ma, 2006)
- Dynamic Multi-stage Stochastic Programming

Survey of tools used for Strategic ALM

(Fabozzi et al. 2005) surveyed 28 defined-benefit pension funds (with a total of USD 436 billion assets under management) in four countries (US, UK, CH, NL). While many funds still prefer simulation to optimization, over 2/3 of the participating funds said that they use optimization, but

> ALM technology is ok, applying it is a whole other story... The models are disconnected from practice - often they are only cosmetic. A lot of time and effort is spent fitting the data into accounting standards rather than adhering to the model.

(Anonymous US pension fund manager)

Multi-stage Stochastic Programming - Advantages

Reasons for using Multi-stage Stochastic Optimization for ALM:

- Incorporates multiple correlated sources of risk for both assets and liabilities,
- supports long time horizons (discrete time stages),
- accommodates risk aversion, supports variety of risk measures,
- allows for dynamic portfolio rebalancing while satisfying operational, and regulatory restrictions, as well as policy requirements,
- provides simple integration of realistic constraints, e.g. transaction costs.

Multi-stage stochastic programming - Main issues

Issue One - Modeling underlying decision problem - Multi-stage models and scenario model (tree) handling are considered to be too complex to be used in companies for real-world applications. Communication of tree-based models to non-experts is complicated.

Issue Two - Modeling underlying uncertainty - A discrete tree approximation of the underlying stochastic process has to be generated in order to numerically compute a solution. The quality of the scenario model severely affects the quality of the solution (garbage in $\rightarrow$ garbage out).

Both issues are valid since the inception of stochastic programming, nowadays a wide range of tools exist (scenario generators and modeling environments)

Survey of selected multi-stage stochastic PFM models

Many models have been proposed, a small subset surveyed to summarize the wide range of modeling structures and parameters:

- US (Mulvey et al., 2000)
- The Netherlands (Kouwenberg, 2001)
- Finland (Pennanen et al., 2005)
- Switzerland (Dondi et al., 2005)

Central element of all four models: dynamic asset allocation

Survey: US (Mulvey et al., 2000)

- Towers Perrin-Tillinghast: Stochastic ALM system for helping its pension plan clients understand the risks and opportunities.
- Success story: US WEST saved USD 450 to USD 1,000 million in opportunity costs in its pension plan.
- Input: Scenarios generated using CAP:Link (Mulvey, 1996) - simulation of key economic variables, such as price and wage inflation, interest rates for different maturities (real and nominal), stock dividend yields and growth rates, and exchange rates through each year for a period of up to 40 years. Returns on asset classes and liability projections are consistent with the underlying economic factors. Common global framework.
- Output: Optimal asset allocation (efficient frontier)

Survey: The Netherlands (Kouwenberg, 2001)

- ORTEC consultants.
- Input: Vector Autoregressive Model with 5 factors: wages, deposits, real estate, bonds, and stocks.
- Objective: Minimize the sum of average contribution rates, and consider the risk aversion of the underlying pension fund manager.
- Output: Efficient frontier of costs vs. downside risk.
- Computational considerations using special decomposition methods to solve realistically large models.

Survey: Finland (Pennanen et al., 2005)

- Ilmarinen Mutual Pension Insurance Company.
- Input: Vector Equilibrium Correction approach (Koivu et al., 2004) for modeling 7 asset and liability values:
  - Assets: Stocks, property, loans.
  - Liabilities: Reserves, tech interest rates, cash flows, solvency capital.
- Model: Convex approximations of (non-convex) regulatory objective and constraints: solvency borders, maximum amount of yearly bonus transfers.
- Output: Asset allocation, which clearly outperforms realistic fixed-mix strategies as well as portfolio insurance strategies (Black and Jones, 1988).

Survey: Switzerland (Dondi et al., 2005)

- Input: Factors (interest rates, earnings-price ratio, momentum, etc) are modelled as linear stochastic, mean reverting processes. Two asset categories: Risky assets, and money market (depending on factors).
- Model: Bucket system: Pooling and splitting of pension funds can be done by adding bucket structures - portfolio optimization done for individual buckets instead of lump sum.
- Objective: Optimal liability adjusted strategy determined by two policies for each bucket:
  - Earmarking strategy (fraction of current net wealth provided for expected payments).
  - Investment strategy (where earmarked wealth is invested).
- Output: Asset allocation.

Step Three

- Build a multi-stage stochastic PFM modeling and optimization system for an Austrian pension fund.

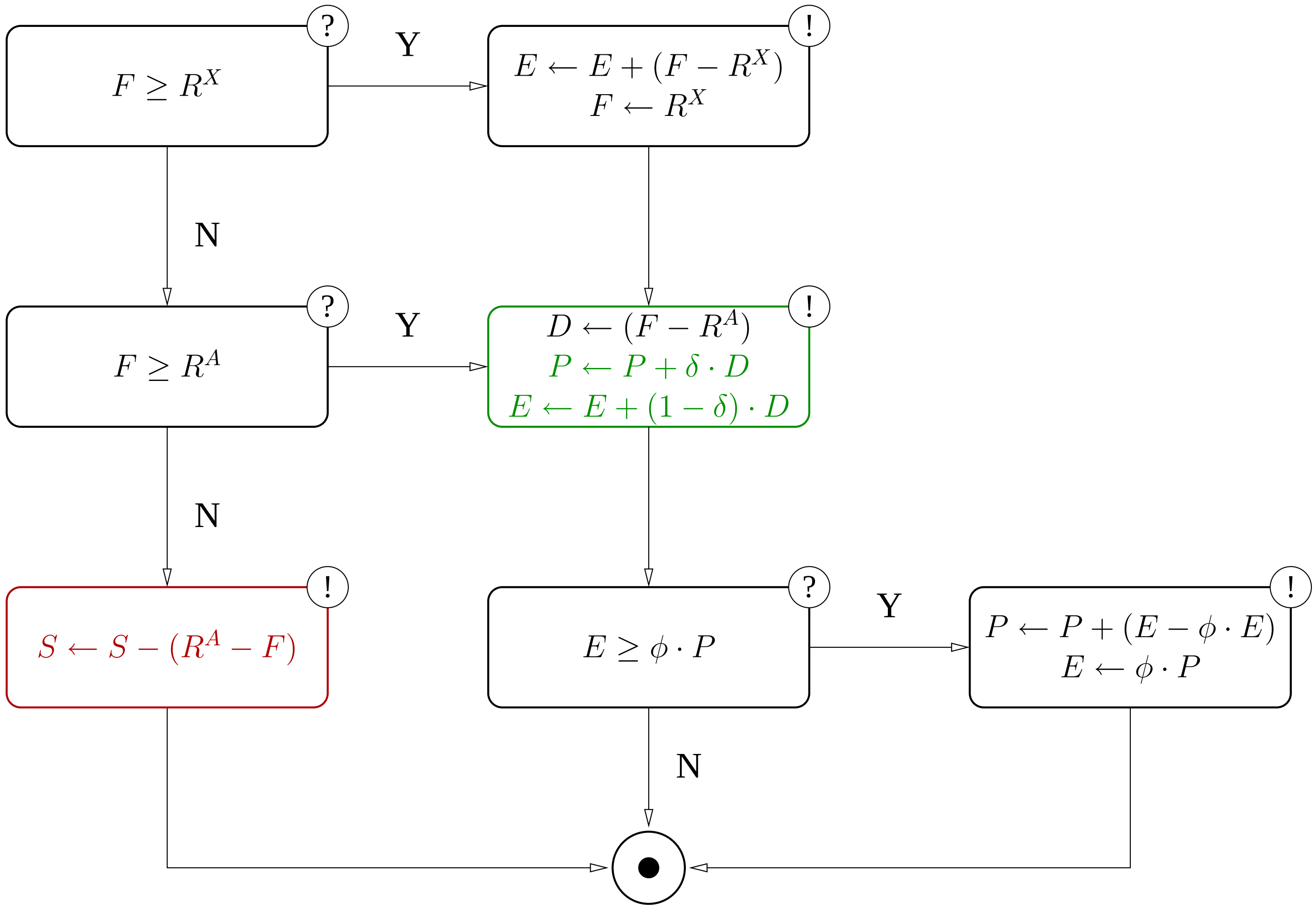
Austrian Pension Fund System

Some facts about Austrian pension regulation (PKG):

- Active and passive members are cumulated into assessment- and risk groups (ARG) consisting of at least 1000 members, cf. PKG §12(2).
- Important: Handling of equalization fund (Schwankungsrückstellung), used to compensate losses from bad asset return results, cf. PKG §24 and §24a.
- Minimum guarantees can be excluded, cf. PKG §2(2).

Management decisions regarding the equalization fund are a main focus for strategic Austrian Pension Fund ALM decision optimization.

Management decisions - Premium and equalization fund



Setting up the optimization model

Deterministic parameters:	
T	Number of stages
$F^{E/P}$	Maximum level of equalization fund relative to premium fund
R^A	Actuarial interest rate
R^X	Excess interest rate
Stochastic parameters:	
ρ_t	Return at stage t
ϕ_t	Cash flow (liabilities) at stage t
Variables:	
p_t	Value of premium fund at stage t
e_t	Value of equalization fund at stage t

Decision: Find optimal management decision vector δ_t regarding the yearly distribution of excess wealth to the premium or equalization fund.

Setting up the optimization model

Objective: Maximize Average Value-at-Risk (AVaR) of e_T at some level α.

(Multi-)Stage-based formulation, stages $\mathcal{T} = (0, \ldots, T)$:

$$
\begin{array}{lll}
\text{maximize} & AVaR(e_T) & \\
\text{subject to} & \rho_t - R^A = d_t^+ - d_t^- & \forall t \in (1, \ldots, T) \\
& p_t = \phi_t \cdot p_{t-1} + p_{t-1} \cdot R^A + \beta_t \cdot p_{t-1} d_t^+ & \forall t \in (1, \ldots, T) \\
& e_t = e_{t-1} + (1 - \beta_t) \cdot p_{t-1} d_t^+ + p_{t-1} d_t^- & \forall t \in (1, \ldots, T) \\
& d_t^+ \geq 0, d_t^- \geq 0 &
\end{array}
$$

Stochastic variables	d_t^+, d_t^+, p_t, e_t
Stochastic parameters	ρ_t, ϕ_t
Deterministic variables	β_t
Parameters	R^A

Setting up the optimization model - AMPL extension

```
stochastic p, e: 0..T;
stochastic dp, dm, cSurplus, cPremium, cEqualization: 1..T;
deterministic beta: 1..T;
stochastic z, cAVaR: T;

var p, e;
var dp >= 0, dm >= 0;
param rho, phi;
param ra, alpha;
var g; var z >= 0;

maximize objFunc: g - ( E(z / ( 1 - alpha ), T) );
subject to cSurplus: rho - ra = dp - dm;
subject to cPremium: p = phi * p(-1) + p(-1) * ra + beta * p(-1) * dp;
subject to cEqualization: e = e(-1) + (1 - beta) * p(-1) * dp + p(-1) * dm;
subject to cAVaR: z >= g - e;
```

Setting up the optimization model - extensions

- Asset management, i.e. replace ρ_t with a multi-dimensional stochastic process of underlying assets and add a decision variable x specifying the portfolio, including common portfolio composition constraints.
- Bi-criteria objective function (Assets and Liabilities).
- Tri-criteria objective function:
  - Maximize asset portfolio return,
  - Minimize asset portfolio risk,
  - Maximize AVaR of equalization fund.
- Dynamic risk measures (e.g. stage-wise AVaR)
- Management rule constraints for β_t, e.g. $|\beta_t - \beta_{t-1}| \leq F^\beta$

Simulation of input parameters

Asset side: Using standard models for simulating the set of model portfolios.

Liability side: Special simulation tools in close cooperation with partner OePAG (Österreichische Pensionskassen AG):

- Using real (historical) data of various assessment- and risk groups.
- Modeling special contract details (e.g. extended support for widows and orphans, special disability features for heavy workers).
- Checking simulation model validity against internal IT systems (VP/MS).

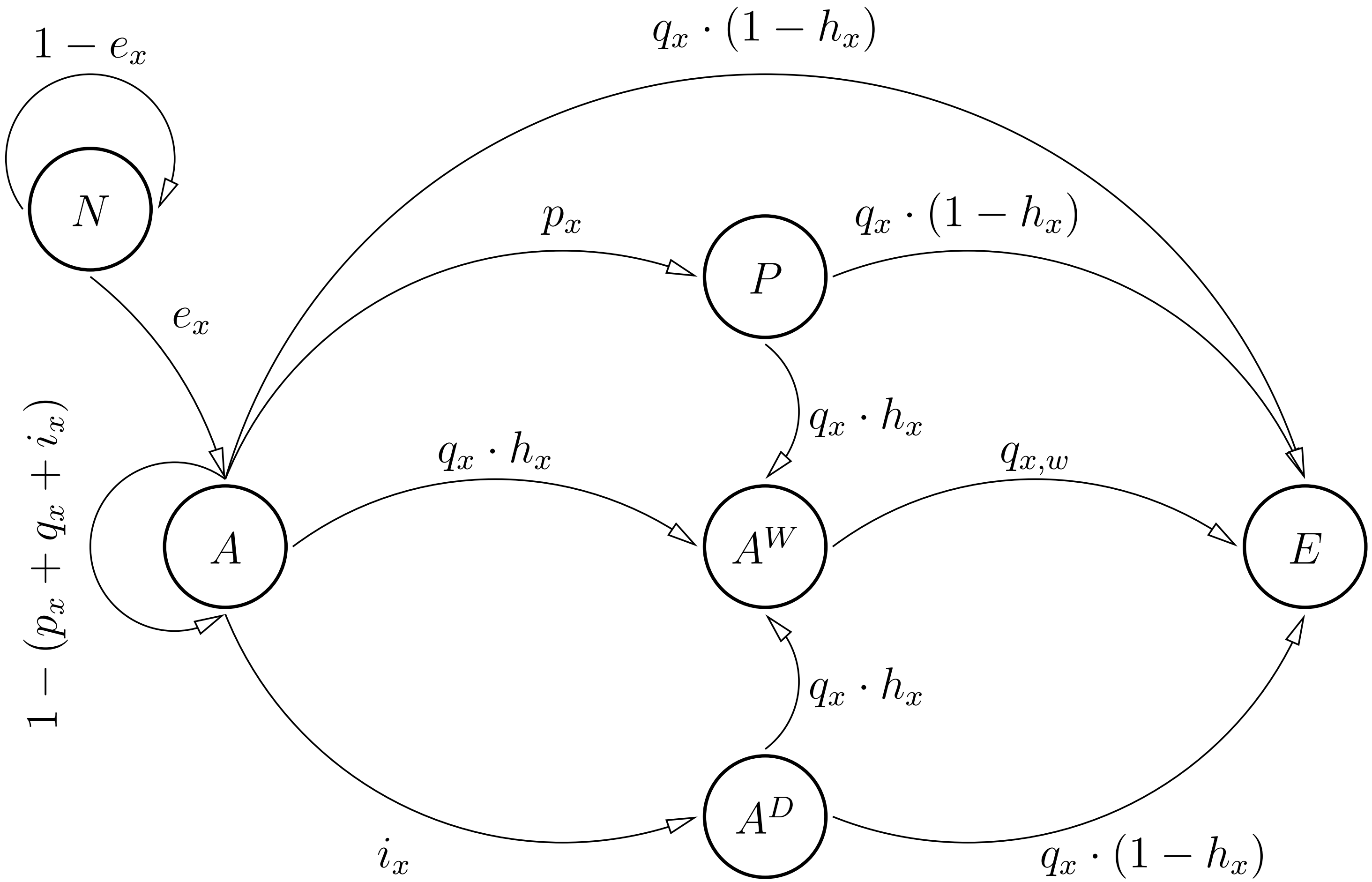
Simulation of the liability side

Simulation of every member of one ARG indiviually - status at each month:

- Main parameters: Age, Gender
- Status (simplified set):
  - $[N]$ot participating,
  - $[A]$ctive,
  - $[A^W]$ctive (widow), $[A^D]$ctive (disabled),
  - $[P]$assive (retired),
  - $[E]$nd (died).

Status $[N]$: shadow participants, entering the ARG at a later point in time.

Transition diagram



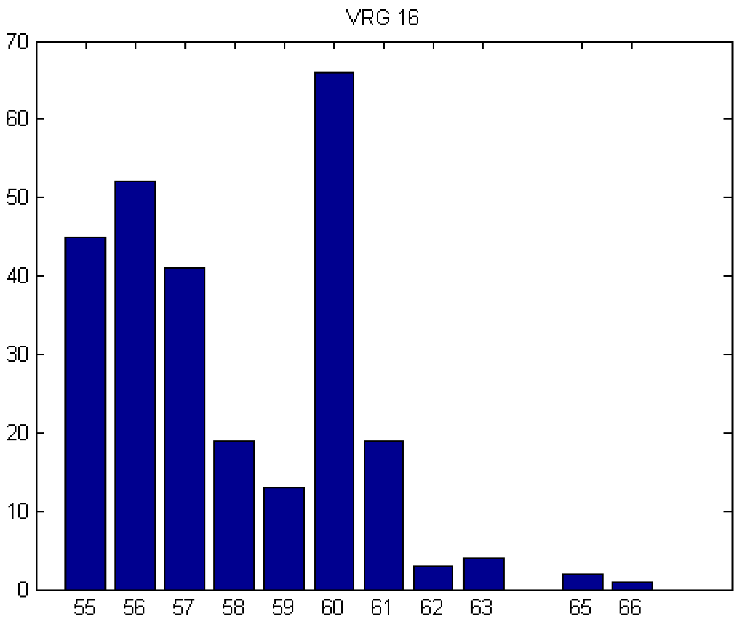
Calculation of transition matrices - Input

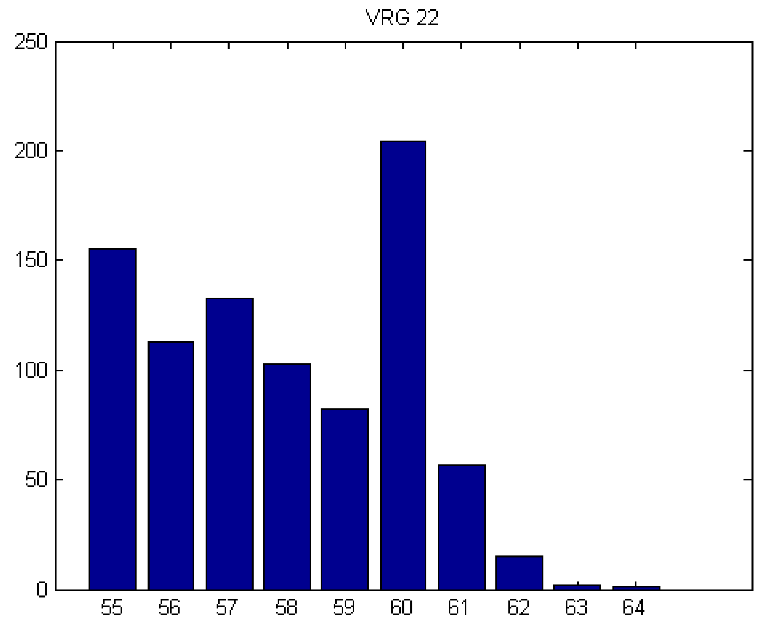
- Austrian mortality tables, with additions: probabilities for becoming disabled and age of widows: q_x, i_x, h_x.

In addition, as (Dondi, 2006) pointed out for future research: Investigation of true entry and exit probabilities for a specific pension fund, rather than using national averages:

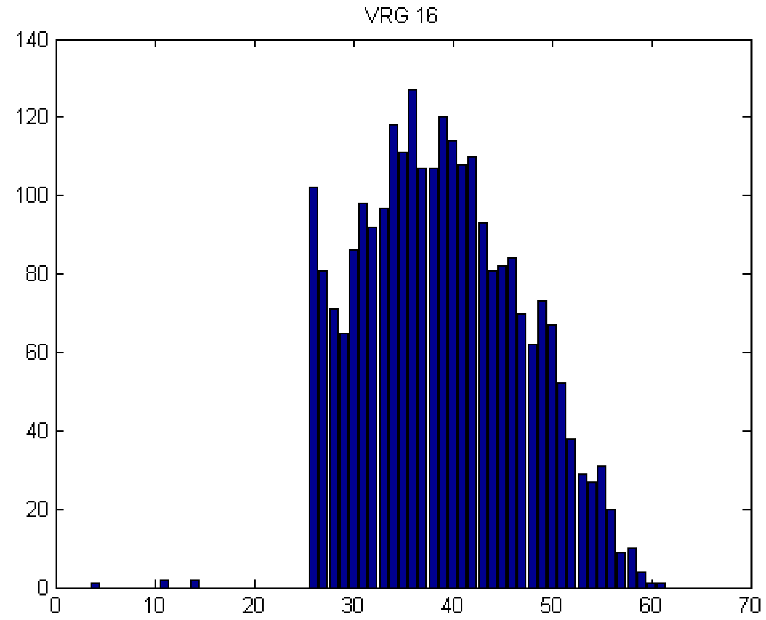
- Probability model for age of retirement (using historical data): p_x.
- Probability model for entering the ARG (fluctuation, using historical data, shadow participant concept): e_x.

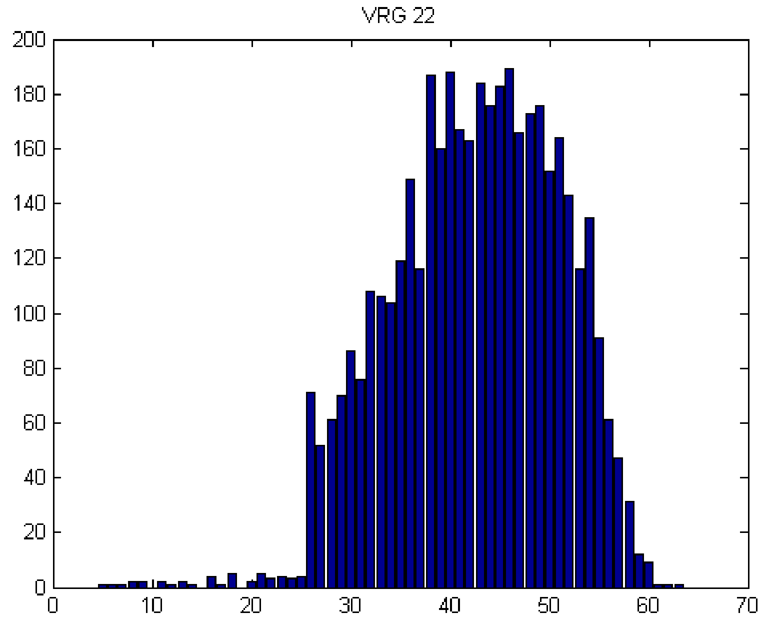
Probability model for age of retirement





Probability model for entering an ARG





Calculation of transition matrices

Output: Transition matrix $Q_{a,g}$ for each age a and gender g.

Usage: Given the current state of an ARG:

- Simulate future development (status, payments, benefits) of each individual.
- Aggregate values at each optimization time stage t (yearly time stages).
- Generate a scenario tree out of aggregated (uni-variate) path.
- Use the aggregated liability scenario tree for the optimization.

(Currently, no explicit correlation between assets and liabilities is considered)

Survey: Austria (CRM, 2007)

- OePAG (Österreichische Pensionskassen AG)
- Input: Detailed liability scenarios stemming from individual ARG population simulation models, model portfolios, Sekundärmarktrendite.
- Objective: Maximize Average Value-at-Risk of equalization fund.
- Model specification: Using a set of model portfolios, and dynamically building linear combinations based on the respective liability status (convex approximations).
- Output: Management strategy concerning optimal budget distribution to premium or equalization fund.

Contact & More Information

Contact Information

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Session Announcement TC4 (Tuesday, 16.00-18.00) Room: Aula

- Kilianova, Sona: Pension planning under multi-period risk minimization.
- Zelle, Hildegard: A modern asset liability management system for Austrian pension funds.
- Streutker, Matthijs: The Importance of Detailed Indexation Modeling in ALM for Dutch Pension Funds.
- Simsek, Koray: Improving Investment Performance for Pension Funds.